Sample AP Calculus Questions from Problem-Attic

1 The Wonder Widget company sells widgets for \$79.99 each. The cost to manufacture widgets is given by the formula $C = 61n + 1050$, where n is the number of widgets sold. For what values of n will the company realize a profit?

a) $n \geq 8$ b) $n \geq 18$ c) $n \geq 33$ d) $n \geq 52$ ∗e) $n \geq 56$

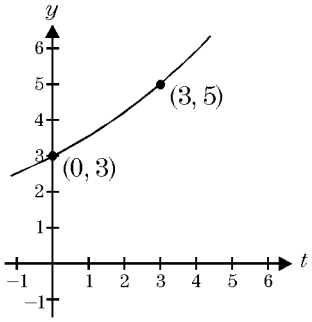
2 Find the exact value of $\cos\left[\arctan(-\frac{2}{3})\right]$.

a) $-\frac{\sqrt{65}}{4}$ ∗b) $\frac{3\sqrt{13}}{13}$ c) $-\frac{2\sqrt{13}}{13}$ d) $\frac{2\sqrt{13}}{13}$ e) $\frac{4\sqrt{5}}{5}$

3 Find the constant k so that the exponential function $y = 3e^{kt}$ passes through the points given on the graph.

∗a) $\frac{1}{3}\ln\frac{5}{3}$ b) $\ln\frac{5}{9}$ c) $\frac{2}{3}\ln\frac{5}{9}$

d) $\frac{1}{3}\ln\frac{5}{9}$ e) $\frac{2}{3}\ln\frac{5}{3}$



4 What is the range of $\frac{x^2}{25} - \frac{y^2}{4} = 1$?

a) Ø ∗b) $y \in \mathbb{R}$ c) $y \leq 5$ d) $|y| \geq 2$ e) $|y| \geq 5$

5 Find the range of $f(x) = \sqrt{3x - 4}$.

a) $\left(-\infty, -\frac{3}{4}\right]$ b) $\left(-\infty, -\frac{3}{4}\right)$ c) $\left[0, \frac{3}{2}\right)$

d) $\left[\frac{4}{3}, \infty\right)$ ∗e) $\left[0, \infty\right)$

6 If $g(f(x)) = 9 - 6x$, $f(x) = 3x - 2$, and $g(x) = ax + b$, then $g(x) =$ _____

a) $21 - 18x$ b) $21 - 12x$ c) $10 - 2x$ d) $10 - x$ ∗e) $5 - 2x$

7 What are the x-values of the points where the graphs of $y = 3\sin^2(4x) - 7$ and $y = 5\sec x$ intersect for $0 \le x < 2\pi$?

8 Answer using one of: EVEN, ODD, or NEITHER.

$f(x) = \dfrac{x^2}{\sqrt{1 + x^5}}$ is _____.

9 Find A so that $\lim\limits_{x \to 2} \dfrac{x^2 + Ax - 10}{x - 2}$ exists.

10 $\lim\limits_{x \to 0} \dfrac{\sin^2 2x}{\sin^2 5x} =$

∗a) $\frac{4}{25}$ b) ∞ c) $\frac{2}{5}$ d) Ø e) $\frac{25}{4}$

11 $\lim\limits_{x \to -\infty} \dfrac{2 - 2^x}{5 - 5^x}$ is

a) 1 b) 2 c) 0 d) $\frac{1}{5}$ ∗e) $\frac{2}{5}$

12 By using your graphics calculator, show that there exists a number such that 1 less than its square is the same as its square root. Between what 2 consecutive integers does this number lie?

13 The functions f and g have the values shown in the table and are differentiable.

If $A = f \cdot g$, then $A'(6) =$

a) 432 b) 0 ∗c) 389 d) −26 e) 6

x	f	f'	g	g'
0	5	1	−7	$\frac{1}{4}$
2	8	3	−5	1
4	14	9	−3	4
6	26	27	−1	16

14 Let $f(x) = (x^2 - 1)^3$. Over what interval is the function decreasing?

a) $(1, \infty)$ ∗b) $(-\infty, 0)$ c) $(0, \infty)$ d) $(-1, 1)$ e) $(-1, 0]$

15 Given $f(x) = \ln(x - 3) - 2\cos x$. Find the first non-negative interval, $[a, b]$, for which Rolle's Theorem applies and find the corresponding value of c.

16 Find a value which satisfies the conclusion of the Mean Value Theorem for Integrals, given:

$$\int_4^{10} \frac{8}{(x-2)^2}\, dx$$

a) $3\sqrt{2}$ b) $4\sqrt{2}$ c) $5\sqrt{3}$ ∗d) 6 e) -2

17 Find all points of inflection: $f(x) = x^3 - 12x$

a) $(0, 0)$, $(\pm\sqrt{12}, 0)$ ∗b) $(0, 0)$ c) $(2, 0)$, $(-2, 0)$

d) $(2, -16)$, $(-2, 16)$ e) $(0, 0)$, $(2, -16)$

18 Find all intervals on which the function $y = 8x^3 - 2x^4$ is concave upward.

a) $(-\infty, 0)$ and $(2, \infty)$ b) $(-\infty, 24)$ and $(48, \infty)$ c) $(-\infty, 2)$ and $(8, \infty)$

∗d) $(0, 2)$ e) $(24, 48)$

19 Given that $f(x) = \int_0^x \sin(t^3)\, dt$ on the closed interval $[0.5, 2]$, then f has a local maximum at $x =$ _____.

20 $\lim_{x \to 0} \dfrac{\cos ax + 1}{\cos bx - 1} =$

21 Differentiate with respect to x: $y = e^{5-(2/x)}$

a) $\dfrac{dy}{dx} = e^{2/x^2}$ b) $\dfrac{dy}{dx} = e^{5-(2/x)}$ ∗c) $\dfrac{dy}{dx} = \dfrac{2}{x^2}e^{5-(2/x)}$

d) $\dfrac{dy}{dx} = -e^{5-(2/x)}$ e) $\dfrac{dy}{dx} = e^{4-(2/x)}$

22 Find the absolute maximum and absolute minimum of f on $(0, 4]$.

$$f(x) = \frac{x^3 + 2x^2 - 9x}{x}$$

a) Max: None, Min: $(4, 60)$ b) Max: $(0, -9)$, Min: $(-1, -10)$

∗c) Max: None, Min: $(1, -6)$ d) Max: $(0, -9)$, Min: $(1, 6)$

e) Max: None, Min: $(-1, -6)$

23 If $f(x) = \dfrac{3x}{\cos x}$, then $f'(2.014) \approx$

a) 23.109 b) 23.518 ∗c) 22.685 d) 23.905 e) 24.157

24 The point $(6, 2)$ lies on the graph of $f(x) = \dfrac{x-4}{x-5}$. Find the slope of a line tangent to the graph at that point.

a) $-\frac{1}{36}$ ∗b) -1 c) 1 d) $-\frac{1}{9}$ e) 2

25 If $f(x) = \dfrac{4x^3}{3} - 8x^2 + 16x + \dfrac{4}{3}$, then the equation of the tangent at the point of inflection is

a) $2x - y + 8 = 0$ b) $2x - y = 0$ c) $y - 2 = 0$

d) $2x - y - 16 = 0$ ∗e) $y - 12 = 0$

26 Find the derivative of $y = \sqrt[3]{x^2 + x}$.

∗a) $\frac{1}{3}(x^2 + x)^{-2/3}(2x + 1)$ b) $\frac{2}{3}(x^2 + x)^{-2/3}(2x - 1)$ c) $\frac{3}{2}(x^2 + x)^{2/3}(2x + 1)$

d) $\frac{x}{3}(x + 1)^{-2/3}(2x + 1)$ e) $\frac{1}{3}(x^2 + x)^{2/3}(2x + 1)$

27 Find $\frac{dy}{dx}$ given $y^2 - 3xy + x^2 = 7$.

a) $\frac{2x + y}{3x - 2y}$ ∗b) $\frac{3y - 2x}{2y - 3x}$ c) $\frac{2x}{3 - 2y}$ d) $\frac{2x}{y}$ e) $\frac{2y - 3x}{3y - 2x}$

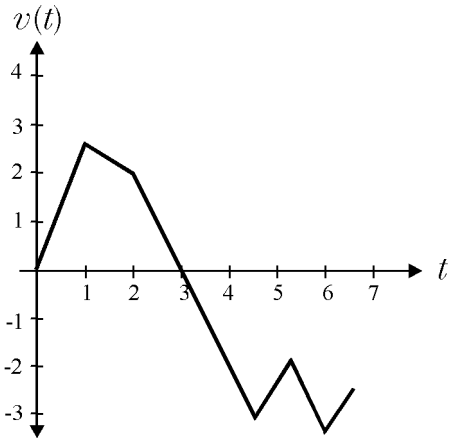
28 In the first quadrant, what is the slope of the tangent line to $x^2 + xy + y^2 = 3$ at the point where $y = 1$?

a) -2 b) -3 c) 3 ∗d) -1 e) 1

29 The graph shows the velocity of a kid in a candy store isle for t on $[0, 6]$.

The object is furthest to the right when $t =$ _____.

a) 1 ∗b) 3 c) 4 d) 6 e) 7



30 The position of road runner at any time t is given by $s = t^3 - \frac{9}{2}t^2 - 12t + 4$. When does $a = 0$?

a) -4, 1 ∗b) 1.5 c) 4, -1 d) 4 only e) 1 only

31 A clown is blowing up a bubble which is in the shape of a sphere. If it is inflated at the rate of $6\,\text{ft}^3/\text{min}$, what is the volume of the balloon when the radius is increasing at the rate of $3\,\text{in/min}$?

a) $\frac{2}{3}\sqrt{\frac{2}{\pi}}\,\text{ft}^3$ b) $\frac{\sqrt{3\pi}}{\pi^3}\,\text{ft}^3$ c) $\frac{\sqrt{2\pi}}{3\pi}\,\text{ft}^3$ ∗d) $8\sqrt{\frac{6}{\pi}}\,\text{ft}^3$ e) $\frac{2\sqrt{2\pi}}{3}\,\text{ft}^3$

32 Find the indefinite integral: $\int \frac{3+4x^{3/2}}{\sqrt{x}}\,dx$

a) $\frac{3}{2}\sqrt{x}+2x^2+C$ b) $-\frac{3}{2}x^{-3/2}+4+C$ c) $\frac{3}{2}x^{-3/2}+2x^2+C$

∗d) $6\sqrt{x}+2x^2+C$ e) $3x^{-1/2}+4x+C$

33 $\int x\sqrt{4-9x^2}\,dx =$

∗a) $-\frac{1}{27}(4-9x^2)^{3/2}+C$ b) $-\frac{1}{18}(4-9x^2)^{3/2}+C$ c) $\frac{3}{2}(4-9x^2)^{3/2}+C$

d) $-\frac{4}{27}(4-9x^2)^{3/2}+C$ e) $\frac{2}{27}(4-9x^2)^{3/2}+C$

34 If $\frac{dy}{dx}=e^{7x}$, then $y=$

a) $7e^{\frac{1}{7}x}+C$ b) $\frac{1}{7}e^{\frac{1}{7}x}+C$ ∗c) $\frac{1}{7}e^{7x}+C$ d) $7e^{7x}+C$ e) $e^{7x}+C$

35 $\int \frac{x}{25+x^4}\,dx =$

a) $\frac{1}{30}\arcsin\frac{x^2}{5}+C$ b) $\frac{1}{2}\arcsin\frac{x^2}{5}+C$ c) $\frac{1}{5}\arctan\frac{x^2}{5}+C$

d) $\frac{1}{10}\text{arcsec}\frac{x^2}{5}+C$ ∗e) $\frac{1}{10}\arctan\frac{x^2}{5}+C$

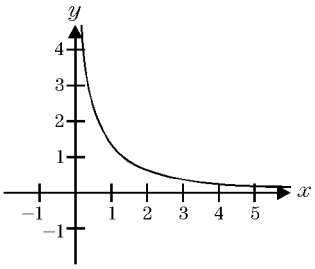
36 Find a four decimal place approximation for $\int_{0.2}^{1}\frac{1}{\sqrt{x}}\,dx$

a) 1.0232 ∗b) 1.1056 c) 1.1471 d) 1.9322 e) 2.4812

37 $\dfrac{d}{dx}\displaystyle\int_2^{x^5} \frac{dt}{t+6} =$

a) $\dfrac{1}{x+6}$ b) $\dfrac{5x^4}{x+7}$ c) $-\dfrac{5x^4}{x^5+6}$ ∗d) $\dfrac{5x^4}{x^5+6}$ e) $\dfrac{x^5}{x^5+6}$

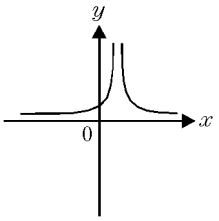
38 The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \le x \le 10$.



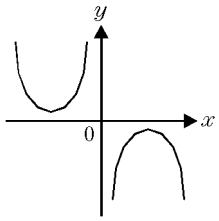
For what value(s) does the function have a relative minimum?

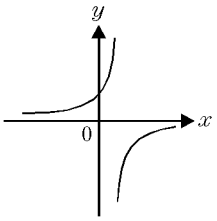
∗a) Ø b) -3 c) 3 d) 0 e) 10

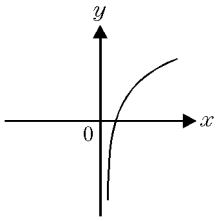
39 The graph of the *derivative* of $f(x)$ is shown here:

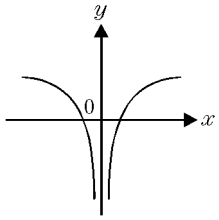


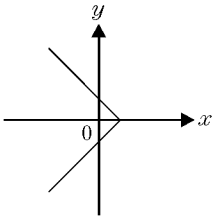
From the following graphs choose f.

a)


∗b)


c)


d)


e)


40 Evaluate: $\int_1^4 xe^{2x}\,dx$

41 Given:

$$\frac{-x^2 + x - 26}{(x^2 + 10)(x - 2)} = \frac{Ax + B}{x^2 + 10} + \frac{C}{x - 2}$$

Decompose the given rational expression to find A, B, and C.

a) $A = -2,\ B = -3,\ C = 2$

b) $A = 2,\ B = 3,\ C = 2$

c) $A = 3,\ B = 0,\ C = -2$

∗d) $A = 1,\ B = 3,\ C = -2$

e) $A = -1,\ B = 3,\ C = -2$

42 Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

∗a) $\frac{2\sqrt{2}}{\pi}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{1}{\pi}$ d) $\frac{\sqrt{2}}{\pi}$ e) $\frac{2}{\pi} - \sqrt{2}$

43 Find the area of the region bounded by $y = 9 - 9x^2$ and $y = 0$.

a) $\frac{19}{3}$ b) $\frac{16}{3}$ ∗c) 12 d) $\frac{25}{3}$ e) $\frac{221}{3}$

44 Find the area above $y = 1$ bounded by $y = 2\sin x$ and $y = 1$, from $x = \frac{\pi}{4}0$ to $x = \frac{\pi}{2}$.

a) $2\sqrt{3}$ units^2 b) $\frac{2\pi}{3}$ units^2 c) $2 + \frac{\pi}{4}$ units^2

∗d) $2 - \frac{\pi}{4}$ units^2 e) $2\sqrt{3} - 2\pi$ units^2

45 A pyramid with a square base and congruent triangular sides is 5 m high. If each cross section of the pyramid is a square parallel to the base, then what is the volume of the pyramid?

∗a) $\frac{125}{3}$ m^3 b) 25 m^3 c) 125 m^3 d) $\frac{25}{3}$ m^3 e) 62.5 m^3

46 Find the volume of the solid formed by revolving the region bounded by $y = \sin x$ and $y = 0$ in the interval $[0, \pi]$ about the x-axis.

a) π^3 ∗b) $\frac{\pi^2}{2}$ c) 2π d) π e) $\frac{3}{2}\pi$

47 A radioactive element has half-life of 50 days. What percentage of the original sample is left after 85 days?

a) 24.06% b) 25.00% c) 28.22% ∗d) 30.78% e) 37.50%

48 Consider the curve given by $x^3y^2 - x^5y = 10$.

a) Show that $\dfrac{dy}{dx} = \dfrac{5x^2y - 3y^2}{2xy - x^3}$

b) Find all points whose x-coordinate is 2 and write an equation for the tangent line at each of these points.

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

49 Find: $\displaystyle\sum_{k=1}^{1000} 5$

a) 5 b) 25 c) 500 d) 1000 ∗e) 5000

50 For any time $t \geq 0$, $x(t) = \sin^2(t)$ and $y(t) = \sin t$. Find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{2}$.

a) -2 b) 2 c) $-\frac{1}{2}$ ∗d) $\frac{1}{2}$ e) π